suggests that policymakers possess an additional instrument of discretionary fiscal policy to control aggregate spending levels.

The conventional model and the modified model

A typical macroeconomic model can be represented by equations (1), (2), and (3) below (see [3]).

\begin{align*}
(1) \quad & y = \bar{c} + \bar{g} + \bar{t} + i + M - \bar{r} + \bar{p} \\
(2) \quad & \bar{r} = \bar{r}(\bar{l}, \bar{K}, \bar{K}) \\
(3) \quad & \bar{p} = \bar{p}(\bar{y}),
\end{align*}

where \( y \) is real income, \( \bar{c} \) is real consumption, \( \bar{i} \) is private real investment demand, \( \bar{r} \) is interest rate, \( \bar{g} \) is government real spending, \( \bar{t} \) is tax rate, \( M \) is nominal money supply, \( \bar{p} \) is price level, and \( \bar{m} \) is real money demand.

The impact of government spending in this model is captured through the single aggregate variable, \( \bar{g} \), which does not interact directly with either private investment or consumption spending. This observation implicitly assumes that private and government spending are independent. A significant portion of government investment spending (e.g., for public investment projects such as roads, airports, and public housing) affects private investment spending directly, and not just through the interest rate. Such government investment spending can decrease, increase, or have no effect on private investment spending. Hence, government investment spending can, respectively, be a substitute for, a complement to, or independent of private investment spending. Similarly, government consumption spending (e.g., for collective consumption goods) can be a substitute for, a complement to, or independent of private consumption spending.

The link between the public and private investment spending can be explained in greater detail. The aggregate production function can be written as:

\begin{align*}
(4) \quad & \bar{y} = \bar{y}(\bar{l}, \bar{K}, \bar{K}) \\
(5) \quad & \bar{y} = \bar{y}(\bar{l}, \bar{K}),
\end{align*}

where \( \bar{y} \) is real output, \( \bar{l} \) is labor, \( \bar{K} \) is the private capital stock, and \( \bar{K} \) is the public capital stock. This is given the quantity of labor and the level of the private capital stock, one could expect the stock of public capital to affect the level of real output. Public and private capital goods can be substitutes, complements, or independent. When the two types of goods are substitutes, an increase in the stock of public capital reduces the marginal productivity of private capital. Conversely, when the two goods are independent, an increase in the stock of public capital increases the marginal productivity of private capital. When the two goods are independent, an increase in the stock of public capital has no impact on the marginal productivity of private capital. An increase in government investment spending that accelerates the formation of public capital therefore affects the marginal productivity of private capital and, thereby, private investment spending because private investment spending depends both upon the marginal productivity of private capital and the interest rate. A change in the marginal productivity relative to the interest rate induces a change in investment spending. The nature of this effect depends upon whether public and private capital goods are substitutes, complements, or independent.

\[ c = \frac{\partial y}{\partial c} \]

\[ \frac{\partial y}{\partial c} = \frac{1}{c} \]

where \( c \) is consumption, \( y \) is output, \( t \) is tax rate, and \( h_i \) is dividend.


2. It is not in this case. This is done partly for convenience, as the conclusions are not affected by the inclusion of a wealth variable in the consumption function, the demand equations. Therefore, the variable is also omitted from the analysis.

3. A referee has suggested that this could have implications for the purchase of public goods ranging from lighthouses to police and fire protection. The consumer product information (12) has considered the case in which some public goods are used as intermediate inputs in private consumption activities.
These observations suggest that equation (1) be reformulated. To allow for both consumption and government spending by the government, one can write:

\[ g = g' + g'' \]

where

- \(g'\) = government spending for current consumption
- \(g''\) = government spending for investment.

If one defines \(x\) as the fraction of total government spending for current consumption, so that \((1 - x)\) is the fraction of total government spending for investment, \(g'\) and \(g''\) can be defined in terms of \(x\) as:

\[ g' = xg \]
\[ g'' = (1 - x)g \]

To allow for more direct interaction between public and private capital in production functions, and for public and private consumption in utility functions, the expressions for investment and consumption \((i)\) are rewritten, respectively:

\[ i = i' + i'' \]
\[ c = c^{'} + c^{''} \]

From conditions (7), (8), and (6), equation (1) can be rewritten:

\[ y = (c^{'} - i^{'} + xg') + (1 - x)g + g'' \]

Solving for \(dy/dt\) using equations (10), (12), and (3) yields:

\[ \frac{dy}{dt} = \frac{1 - (1 - x)\theta + x\phi}{1 - c^{'} - (1 - \theta) + \frac{m_i}{m_0} + \frac{i'M}{m_P} + \frac{P}{P} } \]

where

\[ c^{'} = \frac{dc}{dt} \quad 0 < c^{'} < 1 \]
\[ i^{'} = \frac{di}{dt} \quad 0 < i^{'} < 0 \]

The terms \(i^{'}\) and \(c^{'}\) define to what extent government and private spending are substitutes, independent, or complements. If government investment and/or consumption spending were substitutes for their respective private counterparts, \(c^{'}\) and/or \(i^{'}\) would be negative. If government and private spending behaved as independents, \(i^{'}\) and/or \(c^{'}\) would equal zero. Finally, if government and private spending were complements, \(i^{'}\) and/or \(c^{'}\) would be positive.

Whenever \(i^{'}\) and \(c^{'}\) equal zero: \(x\) equals 1, and \(c^{'}\) equals zero; or \(x\) equals zero, and \(c^{'}\) equals zero; and expression (11) becomes:

\[ \frac{dy}{dt} = \frac{1}{1 - c^{'} - (1 - \theta) + \frac{m_i}{m_0} + \frac{i'M}{m_P} + \frac{P}{P} } \]

This is the textbook value of the multiplier obtained from the standard model. Thus, the results of the conventional model either imply that all government and private spending are independents \((i^{'} = c^{'} = 0)\), or that all government spending is for those items that are independent of their private counterparts \(x = 1\), \(c^{'} = 0; x = 0, i^{'} = 0\).

If the assumption of independence between public and private spending is relaxed, the government spending multiplier differs from that derived in (12). Specifically, \(i^{'}\) is greater than (12) whenever aggregate government spending and private spending are complements and less than (12) when they are substitutes. Whenever both \(i^{'}\) and \(c^{'}\) are negative, the multiplier theoretically could be either zero or negative. The former situation arises when \((1 - x)\theta + x\phi > 1\), and the latter situation arises when \((1 - x)\theta + x\phi < 1\).5

\[ m_i = \frac{dn}{dt} < 0 \]
\[ m_0 = \frac{dn}{dt} > 0 \]
\[ n = \frac{dy}{dt} < 0 \]
\[ P = \frac{dy}{dt} \quad 0 < P \leq 1 \]
\[ i^{'} = \frac{di}{dt} \]
\[ c^{'} = \frac{dc}{dt} \]

5 A referee has suggested that some government purchases fit into neither category e.g., government salary payments to legislators. The introduction of a separate category for such expenditures would not alter this study’s basic results.
possible assuming that either \( i \) or \( c_{1} \) can have a value smaller than \(-1\). Such a result is possible theoretically in simple microeconomic models of consumer choice, for example, it has been shown that a utility-maximizing consumer can respond rationally to certain in-kind subsidies by reducing private purchases by more than the amount of the subsidy.\(^6\)

Policy implications of the modified model

The crowding out controversy

Assumptions about the link between public and private investment spending have important implications for the crowding out controversy. In the standard macroeconomic model, increases in government spending drive up interest rates, thereby discouraging crowding-out private investment. Crowding out is complete when the increase in government spending is offset entirely by a matching decline in private investment spending so that \( d y / d g \) equals zero.

In the standard macroeconomic model described by equations (1), (2), (3), and (12), \( d y / d g \) equals zero only if \( m = \alpha = \beta = 0 \), i.e., if the demand for money is completely interest inelastic, assuming that \( P \) is linear.

In the modified model, the degree of crowding out is partially a function of the relationship between the components on government spending and their private counterparts. This is seen by comparing the multipliers in (11) and (12). If public and private spending were complements, the amount of crowding out in the modified model would be less than in the standard model. Increased government spending would still drive up the interest rate in the modified model, thus discouraging private investment. If public and private spending were complements, they would be offset partially by increased private spending induced directly by increased government spending. If government and private spending were substitutes, the reverse would be true if \( (1 - \alpha - \beta) < -1 \), the multiplier in (11) would equal zero even though \( m \) was not zero. That is, in the modified model, perfect substitutability is a sufficient condition for complete crowding out.

Comparative impact of tax versus spending changes in the modified model

The government tax and expenditure multipliers of the modified model suggest a number of revisions in conventional views of the net impact of balanced budget spending changes. The balanced budget multiplier that emerges from the modified model is:

\[
\frac{dy}{dg} = \frac{(1 - \alpha) \cdot (1 - \beta) + \frac{1}{M} \cdot (i - p)}{1 + \alpha + \beta} \]

When \( h = c_2 = 0 \), the balanced budget multiplier is positive but less than one. This is the standard result. If \( p = 0 \) and private spending are complements, so that \( \beta > 0 \) and \( \alpha > 0 \), the balanced budget multiplier can equal or exceed one. Conversely, if government and private spending are substitutes, so that \( \beta < 0 \) and \( \alpha < 0 \), the balanced budget multiplier can be zero or negative.

This latter result implies that balanced budget reductions in taxes and government spending actually can increase real income. This finding has potentially important implications for fiscal responses to taxpayer revolts. Specifically, the contractionary macroeconomic impacts of equal tax and spending reductions can be reduced if offsetting spending reductions are concentrated among government expenditures that are substitutes for private consumption and investment, rather than independent or complements.

Impact of changing the composition of government spending

The modified model suggests also that changes in the composition of government spending can affect income. This occurs whenever \( \alpha \) and \( \beta \) are unequal. For example, if \( \beta > 0 \) and \( \alpha = 0 \), the impact of government spending on income would depend on the mix of government spending as well as its level. Income would rise (fall) as the share of government spending allocated to private goods substitutes decreased (increased). Even if all items of government spending were substitutes over or complements, changes in the composition of government spending would still affect aggregate demand in various ways.

Recent research by [Marian and Mohnike] suggests that the impact of government spending changes on income would depend on the mix of government spending as well as its level. Income would rise (fall) as the share of government spending allocated to private goods substitutes decreased (increased). Even if all items of government spending were substitutes.
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Table 2: Changes in the Composition of Government Spending

<table>
<thead>
<tr>
<th>Rate of Government Spending</th>
<th>1979</th>
<th>1978</th>
<th>1977</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage Distribution of Government Spending</td>
<td>1979</td>
<td>1978</td>
<td>1977</td>
</tr>
<tr>
<td>Gross Investment</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Nondefense</td>
<td>53.8</td>
<td>54.6</td>
<td>54.4</td>
</tr>
<tr>
<td>Defense</td>
<td>46.2</td>
<td>45.4</td>
<td>45.6</td>
</tr>
<tr>
<td>Leg &amp; Educ</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Health</td>
<td>6.1</td>
<td>2.4</td>
<td>2.7</td>
</tr>
<tr>
<td>Human</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Other</td>
<td>3.1</td>
<td>3.1</td>
<td>3.1</td>
</tr>
</tbody>
</table>

For or complements to private spending, changes in the composition of government spending would still affect income. The degree of substitutability or complementarity differed among varying types of public expenditures. These results provide a theoretical basis for verifying that both the composition and the level of government spending can matter in a macroeconomic sense. A number of interesting implications follow. First, these results suggest that analyzing the impact of fiscal policy solely in terms of the aggregate surplus or deficit may not be appropriate. As shown in Table 2, the composition of government spending between consumption and investment, and between various components of investment, has fluctuated over time. These compositional changes may have affected real income themselves independently of historical changes in the aggregate deficit or surplus. The extent and direction of such compositional changes have yet to be determined empirically. If they are found to be significant, conventional measures of fiscal impact, such as the full employment surplus, would have to be standardized for the mix of government spending as well as for cyclical factors.

A second implication of the model is that decisionmakers may possess additional policy instruments. The modified model suggests that economic activity can be affected by changes in the monetary policy, the aggregate surplus or deficit, and the mix of government spending. Hence, there is one more instrument, not recognized in the conventional analyses, for achieving stabilization targets.

Fiscal policy under full stock-flow equilibrium

The preceding analysis has shown that varying types of public spending are likely to affect aggregate income differently. These results were obtained under a full flow equilibrium. The impact of different types of government spending under a stock-flow equilibrium is examined here. A stock equilibrium is relevant once it is recognized that, from balanced budget changes, increases or decreases in government spending generate expansions or contractions in the money stock and/or the debt amount of government bonds outstanding. These changes in turn generate further impact on income. For example, an increase in government spending that is not financed by a tax increase must be financed either by the sale of bonds or by an expansion of the money stock, which generates further impacts on income.

This relationship between government spending, taxation, and the money stock is demonstrated in terms of the government budget restraint:

\[ g + y = \frac{d}{r} (r + \theta) + \Delta m + \frac{d_0}{r} \]
where

\[ B = \text{number of government bonds, each of which is a perpetuity paying $1 per year.} \]

\[ dB = \text{change in the number of bonds} \]

\[ r = \text{current market price.} \]

\[ dM = \text{change in money stock (or the monetary base in a world with a banking system).} \]

This equation states that government spending must be financed through taxes, bond sales, and/or money creation. Alternatively, if there is a deficit, \( g + B > r(y + B) \), it must be financed by bond issuance and/or money creation.

In order for both stock and flow equilibrium to occur, it is necessary that \( dM = dB = 0 \). If this condition is not satisfied, the stocks of money and/or bonds change, causing wealth and, ultimately, income to change. Thus, in equilibrium, the government budget restraint would become:

\[ (15) \quad g + B = r(y + B) \]

This equation can be used to determine the impact of two different changes in the composition of government spending.

First, consider the effect of changing the mix of \( g \) and \( B \). More specifically, consider a change in the mix of \( g \) and \( B \) such that the total remains unchanged. In this case, it follows that

\[ (16) \quad dB + B = d\left(\frac{g}{y} + B\right) = 0 \]

or

\[ (17) \quad \frac{dy}{d\bar{g}} = 1. \]

Thus, an increase in government spending matched by a decrease in interest payments, holding total spending fixed, increases income by an amount equal to the increase in \( g \). Of course, there is an upper limit to the amount that \( B \) can be decreased.

Determining the impact of different types of exhaustive government expenditures, i.e., the mix of \( g \), is more complicated. In this case, the decisive factor is not whether varying types of public spending differ in their substitutability for or complementarity to private spending. It is rather whether different types of public spending are financed in systematically different ways.

Assume that government spending consists of both consumption and investment spending, with the latter financed by taxes and the latter by bond sales. In this case, equation (15) would be rewritten:

\[ (18) \quad g + B = r(y + B) \]

Solving for \( dy/d\bar{g} \) and \( d\bar{g}/dy \), one obtains:

\[ (19) \quad \frac{dy}{d\bar{g}} = \frac{1}{r} \left( \frac{y + B}{B} \right) dt, \]

and

\[ (20) \quad \frac{d\bar{g}}{dy} = \frac{1}{r} \left( \frac{y + B}{B} \right) dt. \]

According to these equations, increases in \( g \) and decreases in \( B \) increase government spending, which together with positive feedbacks from income and wealth, etc., are important in determining fiscal multipliers. Consequently, changes in government spending affect levels of government revenue, public capital accumulation, and the like.

12. Between fiscal years, federal public debt is doubled.
If \( dy/dg \) and \( dB/dg \), are both positive, then \( dy/dg \) is positive, and \( dB/dg \), is uncertain in sign. It can be positive, negative, or zero. Thus, a bond-financed increase in government investment spending increases income. The impact of a tax-financed increase in government consumption is indeterminate. That is, income can rise, fall, or remain unchanged. 

An interesting implication of these results is that the impact of the changing spending mix can be determined more readily than the impact of changing the spending level. To demonstrate this, one first solves for \( dy/dg \), holding the share of government consumption, \( x \), fixed, and obtains:

\[
(21) \quad \frac{dy}{dg} = \frac{1 - n \frac{dB}{d_g}}{1 + x \frac{dB}{d_g}}
\]

This equation indicates that an increase in total government spending financed by both an increase in the tax rate and bond sales has an indeterminate effect on income. Income can increase, decrease, or remain unchanged.

Holding the level of government spending constant and changing the mix, \( x \), have the following impact on income:

\[
(22) \quad \frac{dy}{dx} = \frac{1 - n \frac{dB}{d_x}}{1 + x \frac{dB}{d_x}}
\]

According to equation (22), an increase in \( x \) decreases income. The reason is that \( dy/dc \) is positive and \( dB/dc \) is negative. Thus, a change in the mix or composition of government spending affects income. Unlike a change in the level of government spending, changing the mix in government spending has an impact on income whose sign can be determined unambiguously on theoretical grounds.

These results have a number of policy implications. Consider first the composition of government spending between goods expenditures (g) and debt interest (4). The ratio of interest payments to government purchases in the federal budget has risen steadily since 1968. If this trend represents a permanent shift in the composition of spending between g and 4, then this analysis implies that this spending shift would lower income. If public consumption and public investment are more likely to be

financed by taxes and debt, respectively, the consequences of increasing government investment spending are more predictable than those of increasing government consumption spending. These results are derived under the assumption that equilibrium exists only when stocks to not change.

Summary and conclusions

The impact of different types of government expenditures on economic activity have been examined. Both flow and stock-flow equilibria have been discussed. In the former, it has been shown that the effect of government spending on income depends largely upon whether government and private spending are compliments, independents, or substitutes, whereas in the latter, it has been shown that the effect of government spending on income depends upon the source of financing. By considering both types of equilibrium, it has been possible to analyze both the sources and uses of government spending. When stocks are not constrained to be constant in equilibrium, the use to which government spending are put has important implications for economic activity, whereas when stocks are constrained to be constant in equilibrium, the sources of government financing have important implications for economic activity. That is, income can be affected by both the composition and level of government spending is the most significant finding. Consequently, policymakers possess another instrument of discretionary fiscal policy in addition to control over aggregate spending levels. Moreover, the empirical assessment of fiscal policy impacts on economic activity may be more complex than suggested by existing macroeconomic models.

REFERENCES
